# MA266 Practice Problems

**1.** If $y' + \left(1 + \frac{1}{t}\right) y = \frac{1}{t}$ and $y(1) = 0$, then $y(\ln 2) =$?

A. $\ln 2 - \ln(\ln 2)$ B. $\ln(\ln 2)$ C. $\ln(\ln 2) + \frac{1}{2\ln 2}$ D. $\frac{1}{\ln 2}\left(1 - \frac{e}{2}\right)$ E. $\frac{1}{\ln 2 - 1}$

**2.** What is the largest open interval for which a unique solution of the initial value problem

$$ty' + \frac{1}{t+1}y = \frac{t-2}{t-3}, \quad y(1) = 0$$

is guaranteed?

A. $0 < t < 1$ B. $0 < t < 2$ C. $0 < t < 3$ D. $-1 < t < 3$ E. $-1 < t < 1$

**3.** An explicit solution of $y' = y^2 - 1$ is?

A. $y = \frac{Ce^{2t}}{1 - Ce^{2t}}$ B. $y = \frac{1 + Ce^{2t}}{1 - Ce^{2t}}$ C. $y = \frac{1}{1 - Ce^{2t}}$ D. $y = \frac{1 + Ce^{2t}}{1 - e^{2t}}$ E. $\frac{y^3}{3} - y = C$

**4.** If $y' = y^3$ and $y(0) = 1$, then $y(-1) =$?

A. $5^{-\frac{1}{4}}$ B. $\sqrt{3}$ C. 1 D. $\frac{1}{\sqrt{3}}$ E. Does not exist

**5.** Let $y(x)$ be the solution to the initial value problem

$$xy' = 3y + 2x^4, \quad y(1) = 0.$$

Then, $y(2)$ is

A. 4 B. 8 C. 16 D. 20 E. 32

**6.** A tank initially contains 40 ounces of salt mixed in 100 gallons of water. A solution containing 4 oz of salt per gallon is then pumped into the tank at the rate of 5 gal/min. The stirred mixture flows out of the tank at the same rate. How much salt is in the tank after 20 minutes?

A. $400 - 360e^{-1}$ B. 20 C. 80 D. $40 + 20e$ E. $400 + 360e^2$

**7.** Find the general solution of a homogeneous equation using substitution $v = \frac{y}{x}$.

$$\frac{dy}{dx} = \frac{5x^2 + 3y^2}{2xy}$$

A. $3y^2 + 5x^2 = Cx^2$ B. $y^2 + 5x^2 = Cx^3$ C. $x^2 + 3y^2 = Cx$ D. $2y - 5x^2 = Cx^4$ E. $y^2 + 3x^2 = Cx^3$

**8.** Suppose that

$$\frac{dy}{dx} = (x+y)^2 - 1.$$

What is the implicit general solution to this differential equation? (Hint: use the substitution $v(x) = x + y$.)

A. $\frac{1}{x+y} - x = C$ B. $\frac{x}{y} + x = C$ C. $\frac{x}{y} - x = C$ D. $x(x+y) + 1 = C$ E. $\frac{1}{x+y} + x = C$

**9.** An implicit solution of

$$y^2 + 1 + (2xy + 1)\frac{dy}{dx} = 0$$

is?

A. $2(xy^2 + y) = C$ B. $xy^2 + y = C$ C. $xy^2 + x + y = C$ D. $\frac{y^3}{3} + y + x^2y + x = C$ E. $y = xy^2 + C$

**10.** Consider the autonomous differential equation

$$\frac{dy}{dt} = -\frac{1}{10}(y-1)(y-4)^2.$$

Classify the stability of each equilibrium solution.

A. $y = 1$ and $y = 4$ both unstable B. $y = 1$ unstable; $y = 4$ stable C. $y = 0$ and $y = 1$ stable; $y = 4$ unstable D. $y = 1$ stable; $y = 4$ semistable E. $y = 0$ stable; $y = 1$ and $y = 4$ unstable

**11.** Consider the following doomsday/extinction differential equation for a population $P(t)$ with the initial population $P(0) = 4$.

$$\frac{dP}{dt} = 3P(P-2)$$

At what time $t$ does "Doomsday" occur (which means the population explodes)?

A. $\frac{\ln(2)}{6}$ B. $\frac{\ln(2)}{3}$ C. $\frac{\ln(4)}{3}$ D. $\frac{\ln(4)}{6}$ E. $\infty$

**12.** Use Euler's method with step size $h = 1$ to find the approximate value of $y(3)$, where $y(x)$ solves the initial value problem

$$y' = x + \frac{y}{2}, \quad y(0) = -8.$$

A. $-17$ B. $-22.5$ C. $-23.5$ D. $-24.5$ E. $-27$

**13.** If the Wronskian $W(f, g) = -3e^{4t}$ and $f(t) = 4e^{2t}$, then $g(t)$ could be

A. $\frac{3}{4}te^{2t}$ B. $12e^{2t}$ C. $-\frac{3}{2}e^{2t}$ D. $-\frac{3}{4}te^{4t}$ E. $-\frac{3}{4}te^{2t}$

**14.** The general solution of

$$y'' - 4y' + 4y = 0$$

is?

A. $y = C_1e^{2t} + C_2te^{2t}$ B. $y = C_1e^{2t} + C_2e^{2t}$ C. $y = C_1e^{2t} + C_2e^{-2t}$ D. $y = C_1e^{-2t} + C_2te^{-2t}$ E. $y = C_1t + C_2t^2$

**15.** The general solution of

$$y''' + 4y'' + 5y' = 0$$

is?

A. $y = C_1e^{-2t}\cos t + C_2e^{-2t}\sin t$ B. $y = C_1 + C_2e^{-2t}\cos t + C_3e^{-2t}\sin t$ C. $y = C_1 + C_2e^t\cos 2t + C_3e^t\sin 2t$ D. $y = C_1 + C_2\cos t + C_3\sin t$ E. $y = C_1 + C_2e^{2t}\cos t + C_3e^{2t}\sin t$

**16.** Let $y(x)$ be the solution to the reducible second-order differential equation

$$y'' + (y')^2 = 0, \ y(0) = 0, \ y'(0) = 1.$$

Find $y(2)$. (Use the substitution $p = y' > 0$.)

A. $\ln 3$ B. $e^{-2}$ C. $\ln 5$ D. $e^4$ E. $4$

**17.** An object weighting 8 pounds attached to a spring will stretch it 6 inches beyond its natural length. There is a damping force with a damping constant $c = 6$ lbs-sec/ft and there is no external force. If at $t = 0$ the object is pulled 2 feet below equilibrium and then released, the initial value problem describing the vertical displacement $x(t)$ becomes?

A. $8x'' + 6x' + 16x = 0, x(0) = -2, x'(0) = 0$ B. $8x'' + 6x' + 16x = 0, x(0) = 2, x'(0) = 0$ C. $\frac{1}{4}x'' + 6x' + 16x = 0, x(0) = 2, x'(0) = 0$ D. $\frac{1}{4}x'' + 6x' + 8x = 0, x(0) = 2, x'(0) = 0$ E. $256x'' + 6x' + 16x = 0, x(0) = 2, x'(0) = 0$

**18.** A particular solution, $y_p$, of

$$y'' - 4y' + 3y = 2t + e^t$$

is?

A. $-\frac{1}{2}te^t + \frac{1}{3}t + \frac{1}{2}$ B. $-\frac{1}{2}te^t + \frac{1}{2}t + \frac{1}{2}$ C. $-\frac{1}{2}e^t + \frac{1}{3}t + \frac{1}{2}$ D. $t^2 + e^t$ E. $-\frac{1}{2}te^t + \frac{2}{3}t + \frac{8}{9}$

**19.** Determine the appropriate form for a particular solution $y_p(x)$ to the third-order differential equation

$$y^{(3)} + y'' - y' - y = \cos x + xe^{-x}.$$

A. $A\cos x + B\sin x + x^2(Cx+D)e^{-x}$ B. $A\cos x + x(Bx+C)e^{-x}$ C. $x^2(A\cos x + B\sin x) + (Cx+D)e^{-x}$
D. $A\cos x + Bxe^{-x}$ E. $A\cos x + B\sin x + (Cx+D)e^{-x}$

**20.** If $y'' + 5y' + 6y = 24e^t$, $y(0) = 0$, $y'(0) = 0$, then $y(1) =$?

A. $e - e^{-2} + 6e^{-3}$ B. $2e - 8e^{-2} + 6e^{-3}$ C. $e - 8e^{-2} + 6e^{-3}$ D. $e + 8e^{-2} + e^{-3}$ E. 0

**21.** The differential equation

$$y'' - \frac{2}{t}y' + \frac{2}{t^2}y = 0$$

has solutions $y_1(t) = t$ and $y_2(t) = t^2$. If

$$y'' - \frac{2}{t}y' + \frac{2}{t^2}y = 2; \quad y(1) = 0,\ y'(1) = 0$$

then $y(2) =$?

A. $8\ln 2 - 4$ B. 0 C. $-6$ D. $8\ln 2 + 4$ E. $8\ln 2$

**22.** A spring-mass system is governed by the initial value problem

$$x'' + 4x' + 4x = 4\cos\omega t$$
$$x(0) = 9, \quad x'(0) = -2.$$

For what value(s) of $\omega$ will resonance occur?

A. 0 B. 2 C. 4 D. no value of $\omega$ E. $2 < \omega < \inf$

**23.** Rewrite the second order equation

$$2u'' + 3u' + ku = \cos 2t$$

as a system of first order equations.

A. $\begin{cases} x' = y \\ y' = \frac{1}{2}(-3x - ky + \cos 2t) \end{cases}$ B. $\begin{cases} x' = x \\ y' = \frac{1}{2}(-3y - kx + \cos 2t) \end{cases}$ C. $\begin{cases} x' = y \\ y' = \frac{1}{2}(-3y - kx + \cos 2t) \end{cases}$
D. $\begin{cases} x' = y \\ y' = 2y + kx + \cos 2t \end{cases}$ E. $\begin{cases} x' = 2y + kx + \cos 2t \\ y' = x \end{cases}$

**24.** The solution of

$$\mathbf{x}' = \begin{bmatrix} 1 & 1 \\ 4 & 1 \end{bmatrix} \mathbf{x}, \quad \mathbf{x}(0) = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

is?

A. $2e^{3t} \begin{bmatrix} 1 \\ 2 \end{bmatrix} + e^{-t} \begin{bmatrix} 1 \\ -2 \end{bmatrix}$ B. $2e^{3t} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + e^{-t} \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ C. $e^{3t} \begin{bmatrix} 2 \\ 1 \end{bmatrix} + e^{-t} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ D. $3e^{3t} \begin{bmatrix} 1 \\ -2 \end{bmatrix} - e^{-t} \begin{bmatrix} 1 \\ -2 \end{bmatrix}$

E. $3e^{3t} \begin{bmatrix} 1 \\ 2 \end{bmatrix} + e^{-t} \begin{bmatrix} 0 \\ -4 \end{bmatrix}$

**25.** Solve

$$\mathbf{x}' = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \mathbf{x}, \quad \mathbf{x}(0) = \begin{bmatrix} -1 \\ 2 \end{bmatrix}.$$

A. $\mathbf{x}(t) = 2e^t \begin{bmatrix} \sin t \\ \cos t \end{bmatrix} - e^t \begin{bmatrix} \cos t \\ \sin t \end{bmatrix}$ B. $\mathbf{x}(t) = 2e^t \begin{bmatrix} \sin t \\ \cos t \end{bmatrix} + e^t \begin{bmatrix} \cos t \\ \sin t \end{bmatrix}$ C. $\mathbf{x}(t) = 2e^t \begin{bmatrix} \sin t \\ \cos t \end{bmatrix} - e^t \begin{bmatrix} \cos t \\ -\sin t \end{bmatrix}$

D. $\mathbf{x}(t) = e^t \begin{bmatrix} \sin t \\ \cos t \end{bmatrix} - e^t \begin{bmatrix} \cos t \\ \sin t \end{bmatrix}$ E. $\mathbf{x}(t) = e^t \begin{bmatrix} -\sin t \\ \cos t \end{bmatrix} - e^t \begin{bmatrix} \cos t \\ \sin t \end{bmatrix}$

**26.** Solve the initial value problem

$$\mathbf{x}' = A\mathbf{x}, \quad \mathbf{x}(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad \text{where } A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}.$$

A. $e^t \begin{bmatrix} 1 \\ 1 \end{bmatrix} - 2te^t \begin{bmatrix} 1 \\ 0 \end{bmatrix}$. B. $e^t \begin{bmatrix} 1 \\ 1 \end{bmatrix} + te^t \begin{bmatrix} 1 \\ 0 \end{bmatrix}$. C. $e^t \begin{bmatrix} 1 \\ 1 \end{bmatrix} + 2te^t \begin{bmatrix} 1 \\ 0 \end{bmatrix}$. D. $e^t \begin{bmatrix} 0 \\ 1 \end{bmatrix} + 2te^t \begin{bmatrix} 1 \\ 0 \end{bmatrix}$. E. $e^t \begin{bmatrix} 1 \\ 1 \end{bmatrix} - 2te^t \begin{bmatrix} 1 \\ 0 \end{bmatrix}$.

**27.** What values of the parameter $\alpha$ in the system below make the origin a saddle point in the phase plane:

$$\mathbf{x}' = \begin{bmatrix} 1 & 1 \\ \alpha & 2 \end{bmatrix} \mathbf{x}$$

A. $\alpha > 2$ B. $\alpha > -\frac{1}{4}$ C. $\alpha < -\frac{1}{4}$ D. $2 > \alpha > -\frac{1}{4}$ E. $\alpha < -2$

**28.** Find a particular solution of

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}' = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} - \begin{bmatrix} 2 \\ 3 \end{bmatrix}.$$

A. $\mathbf{x}_p = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ B. $\mathbf{x}_p = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$ C. $\mathbf{x}_p = \begin{bmatrix} -3 \\ 2 \end{bmatrix}$ D. $\mathbf{x}_p = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$ E. $\mathbf{x}_p = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

**29.** Find the general solution of

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}' = \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} - \begin{bmatrix} 6e^{-t} \\ 1 \end{bmatrix}.$$

A. $c_1 \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^t + c_2 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{2t} + \begin{bmatrix} 2 \\ -1 \end{bmatrix} e^{-t} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

B. $c_1 \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^t + c_2 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{2t}$

C. $c_1 \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^t + c_2 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{2t} - \begin{bmatrix} 6e^{-t} \\ 1 \end{bmatrix}$

D. $c_1 \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^t + c_2 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{2t} + \begin{bmatrix} 6 \\ 0 \end{bmatrix} e^{-t} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

E. $c_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix} e^t + c_2 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{2t} + \begin{bmatrix} 2 \\ -1 \end{bmatrix} e^{-t} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

**30.** $\mathcal{L}\{e^t(1+\cos 2t)\} =?$

A. $\frac{1}{s-1} + \frac{1}{(s-1)^2+4}$ B. $\frac{1}{s-1}\left(\frac{1}{s} + \frac{s-1}{(s-1)^2+4}\right)$ C. $\frac{1}{s-1}\frac{s-1}{s^2-2s+5}$ D. $\frac{1}{s} + \frac{s}{(s-1)^2+4}$
E. $\frac{1}{s-1} + \frac{s-1}{s^2-2s+5}$

**31.** Find the Laplace transform of

$$f(t) = \begin{cases} t, & 0 \le t < 1 \\ 0, & 1 \le t < \infty \end{cases}.$$

A. $e^{-s}\left(\frac{1}{s} + \frac{1}{s-2}\right)$ B. $\frac{1}{s^2} - e^{-s}\frac{1}{s^2}$ C. $\frac{1}{s^2} - e^{-s}\left(\frac{1}{s} + \frac{1}{s^2}\right)$ D. $\frac{1}{s^2} + 2e^{-s}\left(\frac{1}{s} + \frac{1}{s^2}\right)$ E. $e^{-s}\left(\frac{1}{s} + \frac{1}{s^2}\right)$

**32.** Solve

$$y'' + 3y' + 2y = 4u_1(t)$$
$$y(0) = 0, \quad y'(0) = 1.$$

A. $u_1(t)\left(2 - 4e^{-(t-1)} + 2e^{-2(t-1)}\right)$
B. $u_1(t)\left(2 - 4e^{-(t-1)} + 2e^{-2(t-1)}\right) + e^{-t} - e^{-2t}$
C. $u_0(t)\left(2 - 4e^{-(t-1)} + 2e^{-2(t-1)}\right) + e^{-t} - e^{-2t}$
D. $\left(2 - 4e^{-(t-1)} + 2e^{-2(t-1)}\right) + e^{-t} - e^{-2t}$
E. $e^{-t} - e^{-2t}$

**33.** Find the solution of the initial value problem

$$y'' + y = \delta(t-\pi)$$
$$y(0) = 0, \quad y'(0) = 1.$$

A. $y = \sin t + u_0(t)\sin(t-\pi)$ B. $y = \sin t + u_\pi(t)\sin(\pi t)$ C. $y = u_\pi(t)(\sin t + \sin(t-\pi))$ D. $y = u_\pi(t)\sin t$ E. $y = \sin t + u_\pi(t)\sin(t-\pi)$

**34.** The inverse Laplace transform of

$$F(s) = \frac{se^{-s}}{s^2+2s+5}$$

is?

A. $u_1(t)\left(e^{t-1}\cos 2(t-1) - \frac{1}{2}e^{t-1}\sin 2(t-1)\right)$
B. $u_1(t)\left(e^{-t}\cos 2t\right) - \frac{1}{2}e^{-t}\sin 2t$
C. $u_1(t)\left(e^{-t+1}\cos 2(t-1) - \frac{1}{2}e^{-t+1}\sin 2(t-1)\right)$
D. $u_1(t)\left(e^{-t}\cos 2(t-1) - \frac{1}{2}e^{-t}\sin 2(t-1)\right)$
E. $e^{-t+1}\cos 2(t-1) - \frac{1}{2}e^{-t+1}\sin 2(t-1)$

**35.** $\mathcal{L}\left\{\int_0^t \sin 2(t-\tau)\cos(3\tau)d\tau\right\} =?$

A. $\frac{1}{s^2+4} + \frac{s}{s^2+9}$ B. $\frac{2s}{(s^2+4)(s^2+9)}$ C. $\frac{2}{s^2+4} + \frac{s}{s^2+9}$ D. $\frac{2}{(s^2+4)(s^2+9)}$ E. $\frac{s}{(s^2+4)(s^2+9)}$

**Answer Key:**

1. D
2. C
3. B
4. D
5. C
6. A
7. B
8. E
9. C
10. D
11. A
12. C
13. E
14. A
15. B
16. A
17. C
18. E
19. A
20. B
21. A
22. D
23. C
24. A
25. C
26. B
27. A
28. D
29. A
30. E
31. C
32. B
33. E
34. C
35. B